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1991 J. Phys.: Condens. Matter 3 3081

(http://iopscience.iop.org/0953-8984/3/18/003)

J. Phys.: Condens. Matter 3 (1991) 3081–3090. Printed in the UK

Circular dislocation loops in bimaterials

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Received 13 November 1990

Abstract. The solution of the elastic field in bimaterials due to a circular dislocation loop of arbitrary orientation within one of the materials is presented. The bimaterial is idealized as two semi-infinite isotropic elastic solids either perfectly bonded together or in frictionless contact with each other at the planar interface. The solution is obtained simply by integrating the linear superposition of the solutions for the nuclei of strain in a bimaterial over the surface of the cut formally used to generate the dislocation. It is shown that existing solutions for the loop lying in a plane parallel to the interface either inside one of the materials or at the interface between the two semi-infinite solids are special cases of the present general solution.

1. Introduction

The previous studies of a circular dislocation loop for which the elastic fields, energy and Peach–Koehler force can be put into tractable forms are the circular prismatic loop in an isotropic material (Kroupa 1960, Bullough and Newman 1960); the circular glide loop in an isotropic material (Keller 1957 (with corrections given by Kroupa (1962)), Marcinkowski and Sree Harsha 1968), the effects of free surfaces on the circular loop (Chou 1963, Bastecka 1964), the circular prismatic loop lying in a plane parallel to the interface in a bimaterial (Salamon and Dundurs 1971, Dundurs and Salamon 1972), the circular glide loop lying in a plane parallel to the interface in a bimaterial (Salamon and Dundurs 1971, 1977), the circular prismatic loop lying in the interface in a bimaterial (Salamon and Comninou 1979), and the circular glide loop lying in the interface in a bimaterial (Salamon 1981). All these solutions are for the dislocation loop lying on a plane parallel either to the interface of the bimaterial or to the free surface of the half-space. No other loop orientation has been presented.

It is known that, once the Green function for a point force in an elastic body is known, the fields induced in the body by a dislocation can be constructed by integration (Mura 1968). The elastic solution for a force applied at a point in a semi-infinite solid bounded by a plane was given by Mindlin (1936, 1953). The solutions for various nuclei of strain were given by Mindlin and Cheng (1950). The elastic solutions for a point force in joined semi-infinite solids of different elastic properties were presented by Rongved (1955) for the case of perfectly bonded semi-infinite solids, and by Dundurs and Hetenyi (1965) for the case when the joined semi-infinite solids are in frictionless contact with each other. The elastic solutions for various nuclei of strain in two dissimilar media which are

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expressed in terms of Galerkin vectors in a manner analogous to the solutions of Mindlin and Cheng were given by Yu and Sanday (1991a). There, the complexity for the expressions for the displacements and stresses is greatly reduced by expressing the Galerkin vectors in terms of biharmonic and harmonic potential functions.

Tedious calculations are required to pass from the Green function for a point force to the displacement for the infinitesimal loop which are needed (to be integrated over the area of the loop) to obtain the elastic solutions for the dislocation loop. Using this approach and the Green function for the two perfectly bonded half-spaces, Salamon and Dundurs (1971) successfully obtained the solutions for the prismatic and glide loops. Recently, Yu and Sanday (1991b) presented a new method for obtaining the elastic solution of an infinitesimal dislocation loop in a bimaterial. The solutions are obtained by the linear superposition of the solutions for the nuclei of strain. In the present report, the same method is used for obtaining the elastic solutions for a circular dislocation loop with arbitrary orientation in two joined semi-infinite isotropic, elastic solids which are either perfectly bonded or in frictionless contact with each other at the planar interface. The solutions are obtained directly from the integration of the superposition of the Galerkin vectors for the nuclei of strain in the joined semi-infinite solids over the surface of the cut formally used to generate the dislocation.

2. Method of solution

The bimaterial is idealized as two joined semi-infinite isotropic elastic solids: phase I ($x_3 \geqslant 0$) with elastic constants μ and ν, and phase II ($x_3 \leqslant 0$) with elastic constants μ' and ν', which have a common interface $x_3 = 0$. A circular dislocation loop with radius a is located within phase I with its centre at point $(0, 0, c)$. The Burgers vector of the loop is $\boldsymbol{b} = (b_1, b_2, b_3)$, the sign of which is defined by the FS/RH convention, and the surface of the cut formally used to generate the dislocation loop is $\boldsymbol{S} = (S_1, S_2, S_3)$. In the following, the conventional suffix notation will be used, whereby repeated suffixes indicate summation over the values 1, 2, 3, and suffixes following a comma denote differentiation with respect to the Cartesian coordinates corresponding to the indicated suffixes. The elastic field due to the loop is the linear superposition of the field owing to its nine components $b_j S_k$. A component is called a prismatic loop when $j = k$ and a glide loop when $j \neq k$. The boundary conditions at the interface $x_3 = 0$ are, for the perfectly bonded bimaterial,

$$u_i^{\mathrm{I}} = u_i^{\mathrm{II}} \qquad \sigma_{3i}^{\mathrm{I}} = \sigma_{3i}^{\mathrm{II}} \tag{1}$$

and, for the phases in frictionless contact,

$$u_3^{\mathrm{I}} = u_3^{\mathrm{II}} \qquad \sigma_{33}^{\mathrm{I}} = \sigma_{33}^{\mathrm{II}} \qquad \sigma_{3i}^{\mathrm{I}} = \sigma_{3i}^{\mathrm{II}} = 0 \qquad (i = 1, 2) \tag{2}$$

where u_i^{I} and σ_{ij}^{I} are, respectively, the displacements and stresses in phase I, and u_i^{II} and $\sigma_{ij}^{\mathrm{II}}$ are, respectively, the displacements and stresses in phase II.

According to Yu and Sanday (1991b), the Galerkin vector $\boldsymbol{f}^{\mathrm{I}}$ at point $\boldsymbol{r}$ in phase I due to an infinitesimal loop at point $\boldsymbol{r}' = (x_1', x_2', x_3')$ is

$$\boldsymbol{f}^{\mathrm{I}}(\boldsymbol{r}, \boldsymbol{r}') = Db_j \, \mathrm{d}S_k \, (\boldsymbol{g}^{\mathrm{I}}_{jk} + \boldsymbol{g}^{\mathrm{I}}_{kj} - 4\nu \boldsymbol{g}^{\mathrm{I}}_{\mathrm{c}} \delta_{jk}) \tag{3}$$

and the Galerkin vector $\boldsymbol{f}^{\mathrm{II}}$ at point $\boldsymbol{r}$ in phase II is

$$\boldsymbol{f}^{\mathrm{II}}(\boldsymbol{r}, \boldsymbol{r}') = Db_j \, \mathrm{d}S_k \, (\boldsymbol{g}^{\mathrm{II}}_{jk} + \boldsymbol{g}^{\mathrm{II}}_{kj} - 4\nu \boldsymbol{g}^{\mathrm{II}}_{\mathrm{c}} \delta_{jk}) \tag{4}$$

where

$$D = \mu / 8\pi(1 - \nu)$$

$\boldsymbol{g}^{\mathrm{I}}_{\mathrm{c}}$ and $\boldsymbol{g}^{\mathrm{II}}_{\mathrm{c}}$ are the Galerkin vectors in phases I and II, respectively, for the centre of dilatation, and $\boldsymbol{g}^{\mathrm{I}}_{jk}$ and $\boldsymbol{g}^{\mathrm{II}}_{jk}$ are the Galerkin vectors in phases I and II, respectively, either for a double force (when $j = k$) or for a double force with moment (when $j \neq k$). Detailed expressions for the Galerkin vectors for these nuclei of strain can be found in the paper by Yu and Sanday (1991a). By integrating equations (3) and (4) over the surface S_i of the cut, the Galerkin vector $\boldsymbol{F}^{\mathrm{I}}$ at point $\boldsymbol{r}$ in phase I due to a circular loop is

$$\boldsymbol{F}^{\mathrm{I}}(\boldsymbol{r}, \boldsymbol{r}') = Db_j \int_{S_k} (\boldsymbol{g}^{\mathrm{I}}_{jk} + \boldsymbol{g}^{\mathrm{I}}_{kj} - 4\nu \boldsymbol{g}^{\mathrm{I}}_{\mathrm{c}} \delta_{jk}) \, \mathrm{d}S_k \tag{5}$$

and the Galerkin vector $\boldsymbol{F}^{\mathrm{II}}$ at point $\boldsymbol{r}$ in phase II is

$$\boldsymbol{F}^{\mathrm{II}}(\boldsymbol{r}) = Db_j \int_{S_k} (\boldsymbol{g}^{\mathrm{II}}_{jk} + \boldsymbol{g}^{\mathrm{II}}_{kj} - 4\nu \boldsymbol{g}^{\mathrm{II}}_{\mathrm{c}} \delta_{jk}) \, \mathrm{d}S_k. \tag{6}$$

According to the relationships between Galerkin vectors and displacements (Mindlin 1936), one has

$$u^{\mathrm{I}}_i(\boldsymbol{r}) = (1/2\mu)[2(1 - \nu)F^{\mathrm{I}}_{i,jj} - F^{\mathrm{I}}_{k,ki}] \tag{7a}$$

for the point in phase I, and

$$u^{\mathrm{II}}_i(\boldsymbol{r}) = (1/2\mu')[2(1 - \nu')F^{\mathrm{II}}_{i,jj} - F^{\mathrm{II}}_{k,ki}] \tag{7b}$$

for the point in phase II. The stresses are

$$\sigma^{\mathrm{I}}_{ij} = [2\mu\nu/(1 - 2\nu)]u^{\mathrm{I}}_{m,m}\delta_{ij} + \mu(u^{\mathrm{I}}_{i,j} + u^{\mathrm{I}}_{j,i})$$

and

$$\sigma^{\mathrm{II}}_{ij} = [2\mu'\nu'/(1 - 2\nu')]u^{\mathrm{II}}_{m,m}\delta_{ij} + \mu'(u^{\mathrm{II}}_{i,j} + u^{\mathrm{II}}_{j,i}) \tag{8}$$

for the point in phases I and II, respectively. It should be noted that the sequence of integration, in equations (5) and (6), and differentiation in equations (7) and (8) may be interchanged since the integration is done over the coordinate x_i' and the differentiation with respect to the coordinate x_i.

The solution of the elastic field due to a circular dislocation loop of arbitrary orientation in the bimaterial can then be obtained by using equations (5)–(8) together with the Galerkin vectors for different nuclei of strain in the bimaterial given by Yu and Sanday (1991a) (or by integrating the results for an infinitesimal look given by Yu and Sanday (1991b)). It should be noted that, since the solutions are linear superpositions of different nuclei of strain, the boundary conditions, equations (1) and (2), and the supplementary conditions regarding equilibrium, compatibility, and the vanishing of the displacement and stress fields at infinity are automatically satisfied.

3. Circular loop

The Galerkin stress vectors for the double forces, the double forces with moment, and the centre of dilatation in bimaterials given by Yu and Sanday (1991a) are expressed in terms of the biharmonic potential functions R^{I} and R^{II}, the harmonic potential functions φ^{I}, φ^{II}, ϕ^{I} and ϕ^{II}, and the partial derivatives of these functions. These potential functions are

$$\begin{aligned} &R^{\mathrm{I}} = |\boldsymbol{r} - \boldsymbol{r}'| \qquad R^{\mathrm{II}} = |\boldsymbol{r} - \boldsymbol{r}''| \qquad \varphi^{\mathrm{I}} = 1/R^{\mathrm{I}} \qquad \varphi^{\mathrm{II}} = 1/R^{\mathrm{II}} \\ &\phi^{\mathrm{I}} = \log(R^{\mathrm{I}} - z_1) \qquad \phi^{\mathrm{II}} = \log(R^{\mathrm{II}} + z_2) \end{aligned} \tag{9}$$

where

$$\boldsymbol{r}'' = (x'_1, x'_2, -x'_3) \qquad z_1 = x_3 - x'_3 \qquad z_2 = x_3 + x'_3.$$

It has been shown that the integration of the harmonic potentials φ and ϕ over a circular disc S_k can be expressed in terms of the integrals of the Lipschitz–Hankel type involving products of Bessel functions (Gary 1919), and their properties have been considered in detail by Eason *et al* (1955), Salamon and Dundurs (1971) and Salamon and Walter (1979). With the aid of these integrals, the displacements in the two joined semi-infinite solids due to the circular dislocation loop can be obtained as follows.

3.1. *Prismatic loop b_3S_3*

Equations (5) and (6) read, respectively,

$$\begin{aligned} F^{\mathrm{I}} &= 2Db_3 \int_{S_3} (g^{\mathrm{I}}_{33} - 2\nu g^{\mathrm{I}}_{c})\, \mathrm{d}S_3 \\ F^{\mathrm{II}} &= 2Db_3 \int_{S_3} (g^{\mathrm{II}}_{33} - 2\nu g^{\mathrm{II}}_{c})\, \mathrm{d}S_3 \end{aligned} \tag{10}$$

for the bimaterial. By inserting the Galerkin vectors for the centre of dilatation and the double force in the x_3 direction into equation (10), the displacements are found to be, from equation (7), as follows.

(i) For a perfectly bonded bimaterial,

$$\begin{aligned} u^{\mathrm{I}}_i = {} & -[b_3/8\pi(1-\nu)]\{(1-2\nu-\kappa\delta_{3i})\vartheta^{\mathrm{I}}_{3,i} + z_1\vartheta^{\mathrm{I}}_{3,3i} + [1-2(1-\nu)(\mu\beta+\mu'\beta')]\vartheta^{\mathrm{II}}_{3,i} \\ & + (\mu-\mu')\beta[\kappa\delta_{3i}\vartheta^{\mathrm{II}}_{3,i} - x_3\vartheta^{\mathrm{II}}_{3,3i} + (1-2\delta_{3i})\kappa c\vartheta^{\mathrm{II}}_{3,3i} + 2cx_3\vartheta^{\mathrm{II}}_{3,33i}]\} \\ u^{\mathrm{II}}_i = {} & (\mu b_3/4\pi)\{[\beta-(1-2\delta_{3i})\kappa'\beta']\vartheta^{\mathrm{I}}_{3,i} - 2(\beta' x_3 - \beta c)\vartheta^{\mathrm{I}}_{3,3i}\}. \end{aligned} \tag{11}$$

(ii) For a bimaterial in frictionless contact,

$$\begin{aligned} u^{\mathrm{I}}_i = {} & -[b_3/8\pi(1-\nu)]\{(1-2\nu-\kappa\delta_{3i})(\vartheta^{\mathrm{I}}_{3,i} + \vartheta^{\mathrm{II}}_{3,i}) + z_1\vartheta^{\mathrm{I}}_{3,3i} + z_2\vartheta^{\mathrm{II}}_{3,3i} \\ & -2(1-\nu')\mu\alpha[(1-2\nu-\kappa\delta_{3i})(\vartheta^{\mathrm{II}}_{3,i} - c\vartheta^{\mathrm{II}}_{3,3i}) + x_3(\vartheta^{\mathrm{II}}_{3,3i} - c\vartheta^{\mathrm{II}}_{3,33i})]\} \end{aligned} \tag{12}$$

$$u_i^{\rm II} = -(\alpha\mu b_3/4\pi)[(1-2\nu'-\kappa'\delta_{3i})(\vartheta^{\rm I}_{3,i}+c\vartheta^{\rm I}_{3,3i})+x_3(\vartheta^{\rm I}_{3,3i}+c\vartheta^{\rm I}_{3,33i})].$$

In equations (11) and (12),

$$\kappa = 3-4\nu \qquad \kappa' = 3-4\nu' \qquad \beta = 1/(\mu+\kappa\mu') \qquad \beta' = 1/(\mu'+\kappa'\mu)$$

$$\alpha = 1/[(1-\nu')\mu + (1-\nu)\mu']$$

$$\vartheta_3^{\rm i} = \int_{S_3} \varphi^{\rm i}\,{\rm d}S_3 = 2\pi a J_3^{\rm i}(0,1;-1) \qquad ({\rm i}={\rm I},{\rm II})$$

$$J^{\rm i}(m,p;n) = \int_0^\infty J_m(\rho t)J_p(t)t^n\exp(-t|\zeta^{\rm i}|)\,{\rm d}t \qquad (m,n,p \text{ are integers, i} = {\rm I},{\rm II}) \tag{13}$$

$$\rho = (x_1^2+x_2^2)^{1/2}/a \qquad \zeta^{\rm I} = (x_3-c)/a \qquad \zeta^{\rm II} = (x_3+c)/a.$$

The functions J_m and J_p are Bessel functions of the first kind, and order m and p, respectively. The functions $J^{\rm i}(m,p;n)$ are integrals of the Lipschitz–Hankel type. The usual summation convention does not apply in equations (11) and (12). When $c \neq 0$, equation (11) is the same as that given by Salamon and Dundurs (1971) and Dundurs and Salamon (1972). When $c = 0$, equation (11) is the same as that given by Salamon and Comninou (1979) for the dislocation loop at the interface.

3.2. *Prismatic loop b_jS_j, $j = 1$ or 2, and $c > a$*

Equations (5) and (6) read, respectively,

$$F^{\rm I} = 2Db_j\int_{S_j}(g^{\rm I}_{kk}-2\nu g^{\rm I}_{\rm c})\,{\rm d}S_j$$
$$F^{\rm II} = 2Db_j\int_{S_j}(g^{\rm II}_{kk}-2\nu g^{\rm II}_{\rm c})\,{\rm d}S_j. \tag{14}$$

By inserting the Galerkin vectors for the centre of dilatation and the double force in the x_j direction into equation (14), the displacements are found to be, from equation (7), as follows.

(i) For the perfectly bonded bimaterial,

$$\begin{aligned} u_i^{\rm I} = -[b_j/8\pi(1-\nu)][\![\Psi^{\rm I}_{j,jji} &- 2[\nu+2(1-\nu)\delta_{ij}]\vartheta^{\rm I}_{j,i} \\ &+ (\mu-\mu')\beta\{[(1-2\delta_{3i})\kappa + A]\Psi^{\rm II}_{j,jji} + 2x_3\Psi^{\rm II}_{j,jj3i} \\ &- 2\nu\kappa(1-2\delta_{3i})\vartheta^{\rm II}_{j,i} - 4\nu x_3\vartheta^{\rm II}_{j,3i} - 2x_3^2\vartheta^{\rm II}_{j,jji} - A\Theta^{\rm II}_{j,jji} \\ &+ 4(1-2\nu)\delta_{3i}x_3\vartheta^{\rm II}_{j,jj}\} - [4(1-\nu)(\mu-\mu')/(\mu+\mu')]\delta_{ji}\vartheta^{\rm II}_{j,i} \\ &- 2(1-\nu)\mu(\kappa\beta-\kappa'\beta')\delta_{3i}\Phi^{\rm II}_{j,jj}]\!] \end{aligned} \tag{15}$$

$$\begin{aligned} u_i^{\rm II} = -(\mu b_j/4\pi)\{[4/(\mu+\mu')][\Psi^{\rm I}_{j,jji} &- \delta_{1i}\vartheta^{\rm I}_{j,i} + (1-2\nu')(\mu-\mu')\beta'\Theta^{\rm I}_{j,jji}] \\ &- (\kappa\beta+\kappa'\beta')\Psi^{\rm I}_{j,jji} - 4\nu\beta\vartheta^{\rm I}_{j,i} - [(2+\kappa)\beta-(2+\kappa')\beta']\Theta^{\rm I}_{j,jji} \\ &+ 2(\beta-\beta')x_3\Phi^{\rm I}_{j,jji} - (\kappa\beta-\kappa'\beta')\delta_{3i}\Phi^{\rm I}_{j,jj}\}. \end{aligned}$$

(ii) For the bimaterial in frictionless contact,

$$
\begin{aligned}
u_i^{\mathrm{I}} = &-[b_j/8\pi(1-\nu)][\![\Psi^{\mathrm{I}}_{j,jji} + \Psi^{\mathrm{II}}_{j,jji} - 2[\nu + 2(1-\nu)\delta_{ji}](\vartheta^{\mathrm{I}}_{j,i} + \vartheta^{\mathrm{II}}_{j,i}) \\
&- 2(1-\nu')\mu\alpha\{[(1-2\nu)^2 + \kappa\delta_{3i}]\Psi^{\mathrm{II}}_{j,jji} - x_3\Psi^{\mathrm{II}}_{j,jj3i} \\
&+ 2\nu(1-2\nu-\kappa\delta_{3i})\vartheta^{\mathrm{II}}_{j,i} + 2\nu x_3\vartheta^{\mathrm{II}}_{j,3i} - 2(1-2\nu)\delta_{3i}x_3\vartheta^{\mathrm{II}}_{j,jj} \\
&+ x_3^2\vartheta^{\mathrm{II}}_{j,jji} + 2(1-\nu)(1-2\nu)(\delta_{3i}\Phi^{\mathrm{II}}_{j,jj} - \Theta^{\mathrm{II}}_{j,jji})\}]\!]
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
u_i^{\mathrm{II}} = &-(\alpha\mu b_j/4\pi)\{[(1-2\nu)(1-2\nu') + \kappa'\delta_{3i}]\Psi^{\mathrm{I}}_{j,jji} - x_3\Psi^{\mathrm{I}}_{j,jj3i} \\
&+ 2\nu(1-2\nu'-\kappa'\delta_{3i})\vartheta^{\mathrm{I}}_{j,i} + 2\nu x_3\vartheta^{\mathrm{I}}_{j,3i} - 2(1-2\nu')\delta_{3i}x_3\vartheta^{\mathrm{I}}_{j,jj} \\
&+ x_3^2\vartheta^{\mathrm{I}}_{j,jji} - 2(1-\nu')(1-2\nu)\delta_{3i}\Phi^{\mathrm{I}}_{j,jj} + 2(1-\nu)(1-2\nu')\Theta^{\mathrm{I}}_{j,jji} \\
&- 2(\nu-\nu')x_3\Phi^{\mathrm{I}}_{j,jji}\}.
\end{aligned}
$$

In equations (15) and (16),

$$
\begin{aligned}
&A = 2(1-\nu)\mu[(\kappa\beta - \kappa'\beta')/(\mu-\mu')\beta - 4(1-2\nu)/(\mu+\mu')] \\
&\Psi^{\mathrm{i}}_{j,j} = \int_{S_j} R^{\mathrm{i}}_{,j}\,\mathrm{d}S_j = x_j\vartheta^{\mathrm{i}}_j \qquad (\mathrm{i} = \mathrm{I}, \mathrm{II}) \\
&\vartheta^{\mathrm{i}}_j = \int_{S_j} \varphi^{\mathrm{i}}\,\mathrm{d}S_j = 2\pi a J^{\mathrm{i}}(0,1;-1) \qquad (\mathrm{i} = \mathrm{I}, \mathrm{II}) \\
&\Phi^{\mathrm{I}}_j = \int_{S_j} \phi^{\mathrm{I}}\,\mathrm{d}S_j = -\int \vartheta^{\mathrm{I}}_j\,\mathrm{d}x_3 \\
&\Phi^{\mathrm{II}}_j = \int_{S_j} \phi^{\mathrm{II}}\,\mathrm{d}S_j = \int \vartheta^{\mathrm{II}}_j\,\mathrm{d}x_3 \\
&\Theta^{\mathrm{I}}_j = \int_{S_j} z_1\phi^{\mathrm{I}}\,\mathrm{d}S_j = \Psi^{\mathrm{I}}_j + \int \Phi^{\mathrm{I}}_j\,\mathrm{d}x_3 \\
&\Theta^{\mathrm{II}}_j = \int_{S_j} z_2\phi^{\mathrm{II}}\,\mathrm{d}S_j = \Psi^{\mathrm{II}}_j + \int \Phi^{\mathrm{II}}_j\,\mathrm{d}x_3 \\
&J^{\mathrm{i}}(m,p;n) = \int_0^\infty J_m(\rho^{\mathrm{i}}t)J_p(t)t^n \exp\left(-\frac{t|x_j|}{a}\right)\mathrm{d}t \qquad (m, n, p \text{ are integers, } \mathrm{i} = \mathrm{I}, \mathrm{II})
\end{aligned} \tag{17}
$$

$\rho^{\mathrm{I}} = [x_2^2 + (x_3 - c)^2]^{1/2}/a$ for $j = 1$ $\qquad \rho^{\mathrm{I}} = [x_1^2 + (x_3 - c)^2]^{1/2}/a$ for $j = 2$

$\rho^{\mathrm{II}} = [x_2^2 + (x_3 + c)^2]^{1/2}/a$ for $j = 1$ $\qquad \rho^{\mathrm{II}} = [x_1^2 + (x_3 + c)^2]^{1/2}/a$ for $j = 2$.

The usual summation convention does not apply in equations (15) and (16).

3.3. Glide loop b_jS_3, $j = 1$ or 2

Equations (5) and (6) read, respectively,

$$
\begin{aligned}
F^{\mathrm{I}} &= Db_j \int_{S_3} (g^{\mathrm{I}}_{3j} + g^{\mathrm{I}}_{j3})\,\mathrm{d}S_3 \\
F^{\mathrm{II}} &= Db_j \int_{S_3} (g^{\mathrm{II}}_{3j} + g^{\mathrm{II}}_{j3})\,\mathrm{d}S_3.
\end{aligned} \tag{18}
$$

By inserting the Galerkin vectors for the double forces in the x_1 and x_3 directions with the moment about the x_2 axis for $j = 1$, and the double forces in the x_2 and x_3 directions with the moment about the x_1 axis for $j = 2$ into equation (18), the displacements are found to be, from equation (7), as follows.

(i) For perfectly bonded bimaterial,

$$\begin{aligned} u_i^{\mathrm{I}} = -[b_j/8\pi(1-\nu)][\![&-2(1-\nu)\delta_{ji}\vartheta^{\mathrm{I}}_{3,3} - (1-2\nu)\delta_{3i}\vartheta^{\mathrm{I}}_{3,j} + z_1\vartheta^{\mathrm{I}}_{3,ji} \\ &+ (\mu-\mu')\beta[\kappa\delta_{3i}\vartheta^{\mathrm{II}}_{3,j} - x_3\vartheta^{\mathrm{II}}_{3,ji} - (1-2\delta_{3i})\kappa c\vartheta^{\mathrm{II}}_{3,ji} - 2x_3c\vartheta^{\mathrm{II}}_{3,j3i}] \\ &+ 2(1-\nu)\{[(\mu-\mu')/(\mu+\mu')](\delta_{ji}\vartheta^{\mathrm{II}}_{3,3} - \delta_{3i}\vartheta^{\mathrm{II}}_{3,j} + \Phi^{\mathrm{II}}_{3,ji}) \\ &- (\mu\beta - \mu'\beta')\Phi^{\mathrm{II}}_{3,ji}\}]\!] \end{aligned} \tag{19}$$

$$\begin{aligned} u_i^{\mathrm{II}} = -(b_j/4\pi)\{&2\mu\beta\delta_{3i}\vartheta^{\mathrm{I}}_{3,j} - [2\mu/(\mu+\mu')](\delta_{ji}\vartheta^{\mathrm{I}}_{3,3} + \delta_{3i}\vartheta^{\mathrm{I}}_{3,j}) - 2\mu(\beta c - \beta' x_3)\vartheta^{\mathrm{I}}_{3,ji} \\ &+ (1-2\delta_{3i})[(\mu-\mu')/(\mu+\mu') - (\mu\beta - \mu'\beta')]\Phi^{\mathrm{I}}_{3,ji}\}. \end{aligned}$$

(ii) For bimaterial in frictionless contact

$$\begin{aligned} u_i^{\mathrm{I}} = [b_j/8\pi(1-\nu)][\![&2(1-\nu)\delta_{ji}(\vartheta^{\mathrm{I}}_{3,3} - \vartheta^{\mathrm{II}}_{3,3}) + (1-2\nu)\delta_{3i}(\vartheta^{\mathrm{I}}_{3,j} - \vartheta^{\mathrm{II}}_{3,j}) \\ &- z_1\vartheta^{\mathrm{I}}_{3,ji} + z_2\vartheta^{\mathrm{II}}_{3,ji} + 2(1-\nu')\mu\alpha c\{[1-2\nu-2(1-\nu)\delta_{3i}]\vartheta^{\mathrm{II}}_{3,ji} \\ &+ x_3\vartheta^{\mathrm{II}}_{3,j3i}\}]\!] \end{aligned} \tag{20}$$

$$u_i^{\mathrm{II}} = -(\alpha\mu b_j c/4\pi)\{[1-2\nu'-2(1-\nu')\delta_{3i}]\vartheta^{\mathrm{I}}_{3,ji} + x_3\vartheta^{\mathrm{I}}_{3,j3i}\}.$$

In equations (19) and (20),

$$\begin{aligned} \Phi_3^{\mathrm{I}} &= \int_{S_3} \phi^{\mathrm{I}}\,\mathrm{d}S_3 = 2\pi a^2 \frac{|\zeta^{\mathrm{I}}|}{\zeta^{\mathrm{I}}} J^{\mathrm{I}}(0,1;-2) \\ \Phi_3^{\mathrm{II}} &= \int_{S_3} \phi^{\mathrm{II}}\,\mathrm{d}S_3 = -2\pi a^2 \frac{|\zeta^{\mathrm{II}}|}{\zeta^{\mathrm{II}}} J^{\mathrm{II}}(0,1;-2) \end{aligned} \tag{21}$$

and the functions ϑ_3^{I}, $\vartheta_3^{\mathrm{II}}$, J^{I} and J^{II} are as given in equation (13). The unusual summation convention does not apply in equations (19) and (20). When $c \neq 0$, equation (19) is the same as that given by Salamon and Dundurs (1971, 1977). When $c = 0$, equation (19) is the same as that given by Salamon (1981) for the dislocation loop at the interface (note that, owing to a printing error, in Salamon's expression for u_3^{I} the term $\cos(2\theta)$ should read $\cos\theta$).

3.4. *Glide loop b_3S_j, $j = 1$ or 2, and $c > a$*

Equations (5) and (6) read, respectively,

$$\begin{aligned} \boldsymbol{F}^{\mathrm{I}} &= Db_3 \int_{S_1} (g^{\mathrm{I}}_{3j} + g^{\mathrm{I}}_{j3})\,\mathrm{d}S_j \\ \boldsymbol{F}^{\mathrm{II}} &= Db_3 \int_{S_j} (g^{\mathrm{II}}_{3j} + g^{\mathrm{II}}_{j3})\,\mathrm{d}S_j. \end{aligned} \tag{22}$$

By inserting the Galerkin vectors for the double forces in the x_1 and x_3 directions with the moment about the x_2 axis for $j = 1$, and the double forces in the x_2 and x_3 directions with the moment about the x_1 axis for $j = 2$ into equation (22), the displacements are found to be, from equation (7), as follows.

(i) For perfectly bonded bimaterial,

$$\begin{aligned} u_i^{\mathrm{I}} = -[b_3/8\pi(1-\nu)][\![\Psi^{\mathrm{I}}_{j,j3i} - 2(1-\nu)(\delta_{ji}\vartheta^{\mathrm{I}}_{j,3} + \delta_{3i}\vartheta^{\mathrm{I}}_{j,j}) \\ + (\mu-\mu')\beta\{\kappa(2\delta_{3i}-1)\Psi^{\mathrm{II}}_{j,j3i} - 2x_3\Psi^{\mathrm{II}}_{j,j33i} \\ + 4[1-\nu-(1-2\nu)\delta_{3i}]x_3\vartheta^{\mathrm{II}}_{j,ji} + 2x_3^2\vartheta^{\mathrm{II}}_{j,j3i}\} \\ + 2(1-\nu)\{[(\mu-\mu')/(\mu+\mu')](\delta_{ji}\vartheta^{\mathrm{II}}_{j,3} - \delta_{3i}\vartheta^{\mathrm{II}}_{j,j} + \Phi^{\mathrm{II}}_{j,ji}) \\ - (\mu\beta - \mu'\beta')\Phi^{\mathrm{II}}_{j,ji}\}]\!] \end{aligned} \tag{23}$$

$$\begin{aligned} u_i^{\mathrm{II}} = -(b_3/4\pi)\{2\mu\beta\Psi^{\mathrm{I}}_{j,j3i} - [2\mu/(\mu+\mu')](\delta_{ji}\vartheta^{\mathrm{I}}_{j,3} + \delta_{3i}\vartheta^{\mathrm{I}}_{j,j}) \\ - 2\mu(\beta-\beta')x_3\vartheta^{\mathrm{I}}_{j,ji} + (1-2\delta_{3i})[(\mu-\mu')/(\mu+\mu') \\ - (\mu\beta-\mu'\beta')]\Phi^{\mathrm{I}}_{j,ji}\}. \end{aligned}$$

(ii) For bimaterial in frictionless contact

$$\begin{aligned} u_i^{\mathrm{I}} = -[b_3/8\pi(1-\nu)][\![\Psi^{\mathrm{I}}_{j,j3i} - \Psi^{\mathrm{II}}_{j,j3i} - 2(1-\nu)[\delta_{ji}(\vartheta^{\mathrm{I}}_{j,3} - \vartheta^{\mathrm{II}}_{j,3}) \\ + \delta_{3i}(\vartheta^{\mathrm{I}}_{j,j} - \vartheta^{\mathrm{II}}_{j,j})] - 2(1-\nu')\mu\alpha\{(1-2\nu-\kappa\delta_{3i})\Psi^{\mathrm{II}}_{j,j3i} + x_3\Psi^{\mathrm{II}}_{j,j33i} \\ + 2(1-\nu)\delta_{3i}\vartheta^{\mathrm{II}}_{j,j} - 2[1-\nu-(1-2\nu)\delta_{3i}]\vartheta^{\mathrm{II}}_{j,ji} - x_3^2\vartheta^{\mathrm{II}}_{j,j3i}\}]\!] \end{aligned} \tag{24}$$

$$\begin{aligned} u_i^{\mathrm{II}} = (\alpha\mu b_3/4\pi)[(1-2\nu'-\kappa'\delta_{3i})\Psi^{\mathrm{I}}_{j,j3i} + x_3\Psi^{\mathrm{I}}_{j,j33i} + 2(1-\nu')\delta_{3i}\vartheta^{\mathrm{I}}_{j,j} \\ + 2(1-\nu'-\delta_{3i})x_3\vartheta^{\mathrm{I}}_{j,ji} - x_3^2\vartheta^{\mathrm{I}}_{j,j3i}]. \end{aligned}$$

In equations (23) and (24), the functions Ψ_j^{I}, ϑ_j^{I}, Φ_j^{I}, Ψ_j^{II}, $\vartheta_j^{\mathrm{II}}$ and Φ_j^{II} are as given in equation (17). The usual summation convention does not apply in equations (23) and (24).

3.5. *Glide loop b_2S_1 or b_1S_2 and $c > a$*

Equations (5) and (6) read, respectively,

$$\begin{aligned} F^{\mathrm{I}} = Db\int_{S_j}(g^{\mathrm{I}}_{12} + g^{\mathrm{I}}_{21})\,\mathrm{d}S_j \\ F^{\mathrm{II}} = Db\int_{S_j}(g^{\mathrm{II}}_{12} + g^{\mathrm{II}}_{21})\,\mathrm{d}S_j \end{aligned} \tag{25}$$

where $j = 1$ or 2, $b = b_2$ when $j = 1$ and $b = b_1$ when $j = 2$. By inserting the Galerkin vectors for the double forces in the x_1 and x_2 directions with the moment about the x_3 axis into equation (25), the displacements are found to be, from equation (7), as follows.

(i) For perfectly bonded bimaterial,

$$\begin{aligned} u_i^{\mathrm{I}} = -[b/8\pi(1-\nu)][\![\Psi^{\mathrm{I}}_{j,12i} - 2(1-\nu)(\delta_{1i}\vartheta^{\mathrm{I}}_{j,2} + \delta_{2i}\vartheta^{\mathrm{I}}_{j,1}) \\ + (\mu-\mu')\beta\{[(1-2\delta_{3i})\kappa + A]\Psi^{\mathrm{II}}_{j,12i} + 2x_3\Psi^{\mathrm{II}}_{j,123i} + 4(1-2\nu)\delta_{3i}x_3\vartheta^{\mathrm{II}}_{j,12} \\ - 2x_3^2\vartheta^{\mathrm{II}}_{j,12i} - A\Theta^{\mathrm{II}}_{j,12i}\} - [2(1-\nu)(\mu-\mu')/(\mu+\mu')] \\ \times(\delta_{1i}\vartheta^{\mathrm{II}}_{j,2} + \delta_{2i}\vartheta^{\mathrm{II}}_{j,1}) - 2(1-\nu)\mu(\kappa\beta - \kappa'\beta')\delta_{3i}\Phi^{\mathrm{II}}_{j,12}]\!] \end{aligned} \tag{26}$$

$$
\begin{aligned}
u_i^{\mathrm{II}} = -(\mu b/4\pi)\{&[2/(\mu + \mu')][2\Psi^{\mathrm{I}}_{j,12i} - (\delta_{1i}\vartheta^{\mathrm{I}}_{j,2} + \delta_{2i}\vartheta^{\mathrm{I}}_{j,1}) \\
&+ 2(1 - 2\nu')(\mu - \mu')\beta'\Theta^{\mathrm{I}}_{j,12i}] - (\kappa\beta + \kappa'\beta')\Psi^{\mathrm{I}}_{j,12i} \\
&- [(2 + \kappa)\beta - (2 + \kappa')\beta']\Theta^{\mathrm{I}}_{j,12i} + 2(\beta - \beta')x_3\Phi^{\mathrm{I}}_{j,12i} \\
&- (\kappa\beta - \kappa'\beta')\delta_{3i}\Phi^{\mathrm{I}}_{j,12}\}.
\end{aligned}
$$

(ii) For bimaterial in frictionless contact,

$$
\begin{aligned}
u_i^{\mathrm{I}} = -[b/8\pi(1 - \nu)][\![&\Psi^{\mathrm{I}}_{j,12i} + \Psi^{\mathrm{II}}_{j,12i} - 2(1 - \nu)[\delta_{i1}(\vartheta^{\mathrm{I}}_{j,2} + \vartheta^{\mathrm{II}}_{j,2}) \\
&+ \delta_{i2}(\vartheta^{\mathrm{I}}_{j,1} + \vartheta^{\mathrm{II}}_{j,1})] - 2(1 - \nu')\mu\alpha\{[(1 - 2\nu)^2 + \kappa\delta_{3i}]\Psi^{II}_{j,12i} - x_3\Psi^{II}_{j,123i} \\
&- 2(1 - 2\nu)\delta_{3i}x_3\vartheta^{\mathrm{II}}_{j,12} + x_3^2\vartheta^{\mathrm{II}}_{j,12i} + 2(1 - \nu)(1 - 2\nu) \\
&\times (\delta_{3i}\Phi^{\mathrm{II}}_{j,12} - \Theta^{\mathrm{II}}_{j,12i})\}]\!]
\end{aligned}
\qquad (27)
$$

$$
\begin{aligned}
u_i^{\mathrm{II}} = -(\alpha\mu b/4\pi)\{&[(1 - 2\nu)(1 - 2\nu') + \kappa'\delta_{3i}]\Psi^{\mathrm{I}}_{j,12i} - x_3\Psi^{\mathrm{I}}_{j,123i} \\
&- 2(1 - 2\nu')\delta_{3i}x_3\vartheta^{\mathrm{I}}_{j,12} + x_3^2\vartheta^{\mathrm{I}}_{j,12i} - 2(1 - \nu')(1 - 2\nu)\delta_{3i}\Phi^{\mathrm{I}}_{j,12} \\
&+ 2(1 - \nu)(1 - 2\nu')\Theta^{\mathrm{I}}_{j,12i} - 2(\nu - \nu')x_3\Phi^{\mathrm{I}}_{j,12i}\}.
\end{aligned}
$$

In equations (26) and (27) the functions Ψ_j^{I}, ϑ_j^{I}, Φ_j^{I}, Θ_j^{I}, Ψ_j^{II}, $\vartheta_j^{\mathrm{II}}$, Φ_j^{II} and Θ_j^{II} are as given in equation (17). The usual summation convention does not apply in equations (26) and (27).

4. Summary

A general method which is easy to apply has been presented to determine the elastic field in a bimaterial due to a circular dislocation loop from the elastic solution for the nuclei of strain. The bimaterial is idealized as two semi-infinite isotropic elastic solids which may be either perfectly bonded or in frictionless contact with each other at the planar interface. The method consists of obtaining the elastic field due to an infinitesimal dislocation loop by the linear superposition of the Galerkin vectors for the centre of dilatation, double forces and double forces with a moment. The elastic field for a circular dislocation loop can then be obtained by integrating these results over the surface of the cut formally used to generate the dislocation and is expressed in terms of integrals of the Lipschitz–Hankel type. It has been shown that existing solutions for dislocation loops that are parallel either to the interface of the bimaterial or to the free surface of the half-space and that are either inside the solid or the interface, which had previously been obtained through rather tedious and laborious calculations, are special cases of the present general solution. Moreover, problems that heretofore had been intractable, such as in the case for the arbitrary oriented dislocation loop presented, can now be readily solved through the straighforward application of this method.

Acknowledgment

This work has been partially sponsored by the Office of Naval Research through the Naval Research Laboratory.

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